Mathematics Specialist Unit 3

Test 1: Complex Numbers and Polynomial Factorization

**Solutions**

**Student Name:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Section I – calculator-free section**

1. Determine exact expressions in the form for each of the following:
2. (3 marks)

✓

✓

✓

1. (3 marks)

✓

✓

✓

1. The polynomial , where m is a real constant. The equation has a solution of . Determine the value of and the remaining solutions of .

(5 marks)

and is a factor of p(z)

|  |  |  |
| --- | --- | --- |
|  | 6 |  |
|  |  |  |
|  | 30z | 50 | |

and

and the remaining solutions are

1. Two of the solutions to the equation , , are and .
2. State one other solution to the equation. (2 marks)

✓

possible other arguments:

✓

1. Determine the minimum value of and the exact value of in this circumstance.

(3 marks)

max angle between solutions solutions

✓

✓

1. Rewrite in the form for a polynomial of degree 2 and a real number.

(3 marks)

✓

✓

✓

1. Hence, or otherwise, find in the form . (3 marks)

✓

✓

✓

**End of Section I**

**Section II – calculator-assumed section**

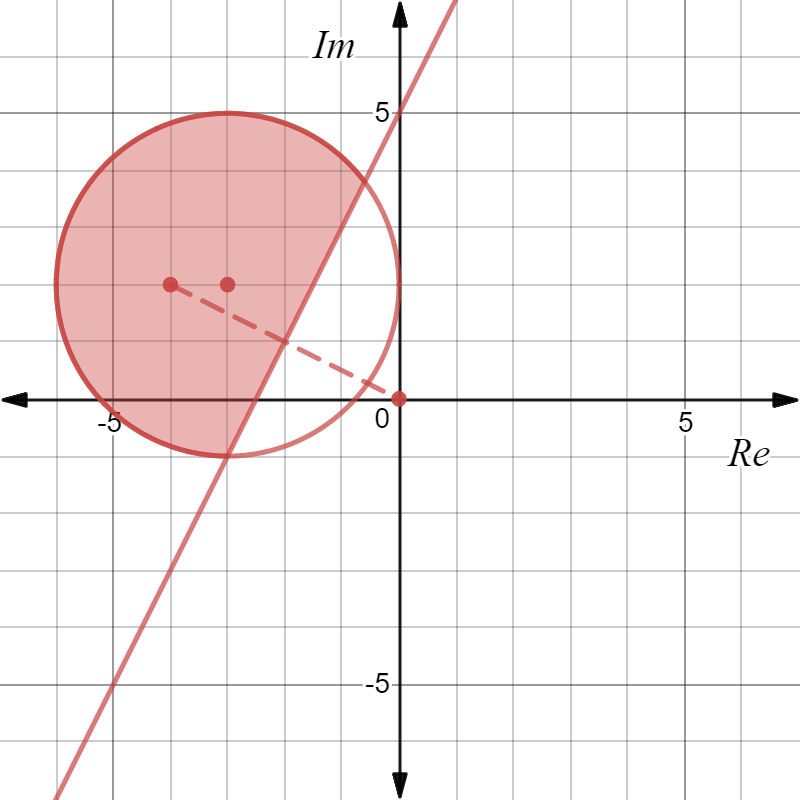
1. Consider the complex equations

and

1. On the set of axes below neatly sketch the graphs of these complex equations

(4 marks)

circle, centre radius 3 and ✓ the perpendicular bisector to the line segment joining to the origin. ✓



✓✓

1. On your diagram shade in the region which satisfies

(2 marks)

Shade inside the circle and above the line. ✓✓

1. Show that (1 mark)

RHS

LHS

1. Hence find all solutions to the equation expressing your answers in the form where and .

(6 marks)

or ✓

or ✓

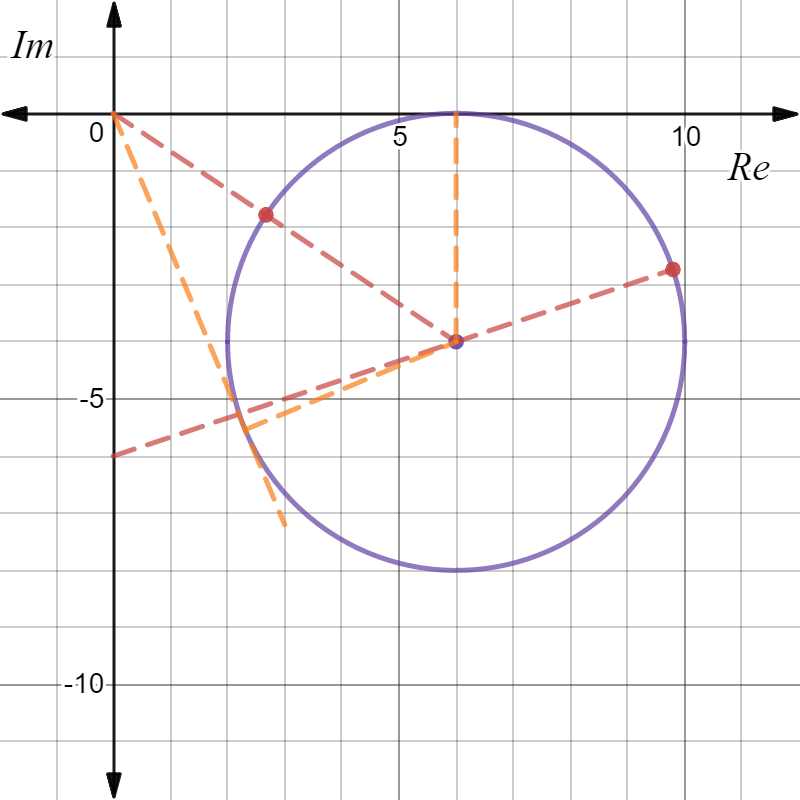
or ✓

or ✓

, ✓

or ✓

1. A subset of points in the complex plane form a circle with centre C and radius 4 units as shown.



**A**

**B**

**C**

1. Mark the position on the plane where is a minimum, clearly label this point A and give this minimum value exactly.

(2 marks)

Min ✓✓

1. Mark the position on the plane where is a maximum, clearly label this point B and give this maximum value exactly.

(2 marks)

Max max distance of from

✓✓

1. Determine to three decimal places the minimum value of .

(3 marks)

Min =radians ( ✓✓✓

1. If and express each of the following in terms of and .
2. (2 marks)

✓

✓

1. (2 marks)

✓

✓

1. (2 marks)

✓

✓

1. Assume .
2. Use De Moivre’s theorem to prove that (3 marks)

LHS

by De Moivre’s theorem ✓

✓

✓

RHS

1. Expand and use the outcome in part (a) above to prove that

. (4 marks)

✓

) ✓

✓

✓

End of Test